Class 12 Maths Chapter 11: Three Dimensional Geometry - Notes, Important Questions and MCQs (2026-27)

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These Class 12 Maths Chapter 11 notes cover “Three Dimensional Geometry” from the NCERT Mathematics Part II textbook (2026-27): direction cosines and direction ratios of a line, vector and Cartesian equations of a line, the angle between two lines, skew lines and the shortest distance between two lines – with formula tables, solved examples, Q&A, 10 MCQs and a free PDF.

Use the table of contents below to jump to the part you need, or read top to bottom for a complete revision of the chapter.

Table of Contents

- Chapter Overview
- Important Topics and Concepts
- Solved Examples
- Line Formulas
- Important Definitions and Terms
- Short Answer Questions
- Long Answer Questions
- MCQs with Answers
- Quick Revision Points
- Chapter Summary
- Exam Tips and PYQ Pattern
- Frequently Asked Questions
- Conclusion

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Chapter Overview

In three dimensions, the direction of a line is described by its direction cosines l, m, n (cosines of the angles with the x, y and z-axes) or by direction ratios a, b, c proportional to them.

A line can be written in vector form $\vec{r} = \vec{a} + \lambda\vec{b}$ or Cartesian form $(x - x_1)/a = (y - y_1)/b = (z - z_1)/c$. Two lines in space may intersect, be parallel or be skew (neither intersecting nor parallel); for non-intersecting lines we find the shortest distance between them.

Important Topics and Concepts

Direction Cosines and Ratios

- If a line makes angles α, β, γ with axes, $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$.

- $l^2 + m^2 + n^2 = 1$.
- Direction ratios a, b, c: $l = a/\sqrt{(a^2 + b^2 + c^2)}$, etc.
- DRs of line through (x_1, y_1, z_1) and (x_2, y_2, z_2): $x_2 - x_1, y_2 - y_1, z_2 - z_1$.

Equation of a Line

- Through point a, parallel to b: $r = a + \lambda b$ (vector).
- Cartesian: $(x - x_1)/a = (y - y_1)/b = (z - z_1)/c$.
- Through two points a and b: $r = a + \lambda(b - a)$.
- Cartesian through two points: $(x - x_1)/(x_2 - x_1) = (y - y_1)/(y_2 - y_1) = (z - z_1)/(z_2 - z_1)$.

Angle between Two Lines

- $\cos\theta = |b_1 \cdot b_2| / (|b_1||b_2|)$.
- Cartesian: $\cos\theta = |a_1a_2 + b_1b_2 + c_1c_2| / (\sqrt{(a_1^2 + b_1^2 + c_1^2)}\ \sqrt{(a_2^2 + b_2^2 + c_2^2)})$.
- Perpendicular: $a_1a_2 + b_1b_2 + c_1c_2 = 0$.
- Parallel: $a_1/a_2 = b_1/b_2 = c_1/c_2$.

Skew Lines and Shortest Distance

- Skew lines: neither parallel nor intersecting; lie in different planes.
- For $r = a_1 + \lambda b_1$ and $r = a_2 + \mu b_2$: $d = |(b_1 \times b_2) \cdot (a_2 - a_1)| / |b_1 \times b_2|$.
- If $d = 0$, the lines intersect.
- Cartesian form uses a 3×3 determinant of $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$ and the two DR rows, divided by $\sqrt{}$(sum of squares of the cross-product components).

Distance between Parallel Lines

- For $r = a_1 + \lambda b$ and $r = a_2 + \mu b$: $d = |b \times (a_2 - a_1)| / |b|$.
- Parallel lines are coplanar.
- Shortest distance is the length of the common perpendicular.

Solved Examples

Board-pattern practice.

- DCs of a line with DRs 2, –1, –2. $\sqrt{(4 + 1 + 4)} = 3$; DCs 2/3, –1/3, –2/3.
- Equation of line through (5, 2, –4) parallel to $3\hat{i} + 2\hat{j} - 8\hat{k}$. $r = 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$; Cartesian $(x - 5)/3 = (y - 2)/2 = (z + 4)/(-8)$.
- Angle between lines with DRs $(1, 1, 2)$ and $(\sqrt{3} - 1, -\sqrt{3} - 1, 4)$. $\cos\theta = |\sqrt{3} - 1 - \sqrt{3} - 1 + 8|/(\sqrt{6}\ \sqrt{24}) = 6/12 = 1/2 \Rightarrow \theta = \pi/3$.
- Shortest distance between $r = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $r = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$. $b_1 \times b_2 = -3\hat{i} + 3\hat{k}$, $a_2 - a_1 = \hat{i} - 3\hat{j} - 2\hat{k}$; $d = |-3 - 6|/\sqrt{18} = 9/(3\sqrt{2}) = 3/\sqrt{2}$.

Line Formulas

Concept	Vector form	Cartesian form
Line through point, parallel to b	$r = a + \lambda b$	$(x - x_1)/a = (y - y_1)/b = (z - z_1)/c$
Line through two points	$r = a + \lambda(b - a)$	$(x - x_1)/(x_2 - x_1) = \ldots$
Angle	$\cos\theta = \lvert b_1 \cdot b_2\rvert/(\lvert b_1\rvert\lvert b_2\rvert)$	$\lvert a_1a_2 + b_1b_2 + c_1c_2\rvert/(\ldots)$
Shortest distance (skew)	$\lvert(b_1 \times b_2) \cdot (a_2 - a_1)\rvert/\lvert b_1 \times b_2\rvert$	Determinant form

Concept	Vector form	Cartesian form
Distance (parallel)	$\|b \times (a_2 - a_1)\|/\|b\|$	—

Important Definitions and Terms

Term	Meaning
Direction cosines	Cosines of angles a line makes with axes
Direction ratios	Numbers proportional to DCs
Skew lines	Non-parallel, non-intersecting lines
Coplanar lines	Lines in the same plane
Shortest distance	Length of common perpendicular
Vector equation	$r = a + \lambda b$
Cartesian equation	Symmetric form of a line
Parameter	λ or μ in line equation
Parallel lines	Proportional direction ratios
Perpendicular lines	$a_1a_2 + b_1b_2 + c_1c_2 = 0$

Short Answer Questions

Q1. What is $l^2 + m^2 + n^2$?

1.

Q2. Write the vector equation of a line through a parallel to b.

$r = a + \lambda b$.

Q3. Find DRs of the line through (1, 2, 3) and (4, 6, 8).

3, 4, 5.

Q4. When are two lines perpendicular?

When $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

Q5. What are skew lines?

Lines that are neither parallel nor intersecting.

Q6. If shortest distance is zero, what does it mean?

The lines intersect.

Q7. DCs of the x-axis?

1, 0, 0.

Q8. When are two lines parallel?

When their direction ratios are proportional.

Long Answer Questions

Q1. Find the shortest distance between the lines (x + 1)/7 = (y + 1)/(–6) = (z + 1)/1 and (x – 3)/1 = (y – 5)/(–2) = (z – 7)/1.

$a_1 = (–1, –1, –1)$, $b_1 = (7, –6, 1)$; $a_2 = (3, 5, 7)$, $b_2 = (1, –2, 1)$.

$b_1 \times b_2 = (–4, –6, –8)$.

$a_2 – a_1 = (4, 6, 8)$.

$(b_1 \times b_2)\cdot(a_2 – a_1) = –16 – 36 – 64 = –116$; $|b_1 \times b_2| = \sqrt{116}$.

$d = 116/\sqrt{116} = 2\sqrt{29}$ units.

Q2. Find the angle between $r = 3\hat{i} + 2\hat{j} – 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $r = 5\hat{i} – 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$.

$b_1 = \hat{i} + 2\hat{j} + 2\hat{k}$, $b_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$.

$b_1\cdot b_2 = 3 + 4 + 12 = 19$.

$|b_1| = 3$, $|b_2| = 7$.

$\cos\theta = 19/21$.

$\theta = \cos^{-1}(19/21)$.

Q3. Find the distance between parallel lines $r = \hat{i} + 2\hat{j} – 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $r = 3\hat{i} + 3\hat{j} – 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$.

$b = 2\hat{i} + 3\hat{j} + 6\hat{k}$, $|b| = 7$.

$a_2 – a_1 = 2\hat{i} + \hat{j} – \hat{k}$.

$b \times (a_2 – a_1) = –9\hat{i} + 14\hat{j} – 4\hat{k}$.

$|...| = \sqrt{(81 + 196 + 16)} = \sqrt{293}$.

$d = \sqrt{293}/7$ units.

MCQs with Answers

1. DCs of y-axis(a) 1, 0, 0 (b) 0, 1, 0 (c) 0, 0, 1 (d) 1, 1, 1Answer: (b) 0, 1, 0

2. $l^2 + m^2 + n^2$ =(a) 0 (b) 1 (c) 3 (d) 2Answer: (b) 1

3. Lines with proportional DRs are(a) perpendicular (b) parallel (c) skew (d) intersectingAnswer: (b) parallel

4. Shortest distance zero ⇒ lines(a) skew (b) intersect (c) parallel (d) perpendicularAnswer: (b) intersect

5. Vector equation of a line(a) $r = a + \lambda b$ (b) $r\cdot n = d$ (c) $x + y + z = 1$ (d) $r = \lambda a$ onlyAnswer: (a) $r = a + \lambda b$

6. DRs of line through (0,0,0) and (1,2,3)(a) 1, 2, 3 (b) 0, 0, 0 (c) 3, 2, 1 (d) 1, 1, 1Answer: (a) 1, 2, 3

7. Perpendicular condition(a) $a_1/a_2 = b_1/b_2$ (b) $a_1a_2 + b_1b_2 + c_1c_2 = 0$ (c) $a_1 + a_2 = 0$ (d) $b_1 \times b_2 = 0$Answer: (b) $a_1a_2 + b_1b_2 + c_1c_2 = 0$

8. Skew lines are(a) coplanar (b) non-coplanar (c) parallel (d) intersectingAnswer: (b)

9. DCs from DRs 2, 3, 6(a) 2/7, 3/7, 6/7 (b) 2, 3, 6 (c) 1/2, 1/3, 1/6 (d) 2/11, 3/11, 6/11Answer: (a) 2/7, 3/7, 6/7

10. Distance between parallel lines uses(a) $b \times (a_2 - a_1)$ (b) $b \cdot (a_2 - a_1)$ (c) $a_1 \times a_2$ (d) $b_1 + b_2$Answer: (a) $b \times (a_2 - a_1)$

Quick Revision Points

- DCs and DRs; $l^2 + m^2 + n^2 = 1$.
- Line equations: vector and Cartesian.
- Angle between lines.
- Parallel and perpendicular conditions.
- Skew lines: shortest distance formula.
- Distance between parallel lines.

Chapter Summary

Lines in space are described by direction cosines (with $l^2 + m^2 + n^2 = 1$) or direction ratios, and written in vector form $r = a + \lambda b$ or Cartesian symmetric form. The angle between lines comes from the dot product of their direction vectors.

Lines in space can be parallel, intersecting or skew. The shortest distance between skew lines is $|(b_1 \times b_2) \cdot (a_2 - a_1)| / |b_1 \times b_2|$, and for parallel lines it is $|b \times (a_2 - a_1)| / |b|$.

Exam Tips and PYQ Pattern

- Convert Cartesian to vector form first.
- Compute cross products carefully.
- State whether lines intersect when $d = 0$.
- Normalise DRs to get DCs.
- Practise shortest distance questions.

Frequently Asked Questions

Is this chapter in the 2026-27 CBSE syllabus?

Yes. It is part of the NCERT Class 12 Mathematics Part II textbook, prescribed in the CBSE 2026-27 syllabus.

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Yes. Use the free PDF link at the top of this page – no signup needed.

How do you find the shortest distance between skew lines?

Use $d = |(b_1 \times b_2) \cdot (a_2 - a_1)| / |b_1 \times b_2|$ for lines $r = a_1 + \lambda b_1$ and $r = a_2 + \mu b_2$.

What are direction cosines?

The cosines of the angles a line makes with the positive x, y and z axes; they satisfy $l^2 + m^2 + n^2 = 1$.

Conclusion

Three Dimensional Geometry gives sure marks in shortest-distance questions. Practise here and save the free PDF.

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Author Bio: Written and reviewed by the Nextoper Editorial Team, a group of CBSE maths educators who build chapter notes directly from the NCERT textbook and past board paper patterns.

Freshness Note: These notes follow the NCERT Class 12 Maths textbook, Mathematics Part II (NCERT reprint 2026-27, rationalised syllabus), and the CBSE syllabus for the 2026-27 session. Last checked on 26 November 2026.